PINCOCK Perspectives

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CALENDAR

- ***25th Annual Mineral Exploration Roundup***
  January 28 –31, 2008
  Westin Bayshore Resort & Marina
  Vancouver, B.C., Canada
  Email: cgirouxschmidt@amebc.ca

- ***110th Annual National Western Mining Conference & Exhibition***
  February 12 –14, 2008
  Westin Hotel
  Denver, Colorado
  Email: jcolgan@coloradomining.org

- ***2008 SME Annual Meeting & Exhibit***
  February 24 –27, 2008
  Salt Palace Convention Center
  Salt Lake City, Utah
  Email: cs@smenet.org
  Visit PAH/Runge at Booths 717 / 719

- ***PDAC 2008 International Convention, Trade Show & Investors Exchange***
  March 2 – 5, 2008
  Metro Toronto Convention Centre
  Toronto, Ontario, Canada
  Email: info@pdac.ca
  Visit PAH/Runge at Booth 708

- ***ExpoMIN 2008 – The Center for the World Mining Industry in Chile***
  April 15 – 18, 2008
  Centro de Eventos Espacio Riesco
  Santiago, Chile
  Email: info@fisa.cl

- ***CIM Conference and Exhibition 2008***
  May 4 – 7, 2008
  Shaw Conference Centre
  Edmonton, Alberta, Canada
  Email: cmurphy@cim.org
  Visit PAH/Runge at Booth 0101

About Kriging

Introduction

Kriging is a geostatistical interpolation technique used in mining industry for interpolation of input point data and estimation of a block model (mineral resource model). The name “kriging” was given by Prof. Georges Matheron in honor of the South African mining engineer Danie Krige.

Kriging is an optimal spatial regression technique which requires a spatial statistical model, popularly known as a variogram, representing the internal spatial structure of the data. The input data (used to estimate any particular block) are weighted based on the variogram model, search parameters and the number of samples used to krige or estimate a block.

A variety of kriging techniques are used in the mining industry. The most common kriging techniques are variants of ordinary kriging, grouped as “linear kriging” techniques. The more complex kriging techniques, such as indicator kriging, disjunctive kriging, etc. are based on non-linear transformation of grades and are grouped as “non-linear kriging” techniques. This article will discuss various aspects of the linear kriging (henceforth “kriging”) techniques.

Why Krige?

Kriging is known by the acronym BLUE: or, the **B**est **L**inear **U**nbiased **E**stimator. In the kriging process the sum of weights assigned to the input data is adjusted to one, and the error of estimation is kept to a minimum. Kriging utilizes the variogram, which does not depend on the actual value of the variable (data), rather its spatial distribution and internal spatial structure. A variogram provides penetrating insight concerning the data used to construct a kriging interpolation system. When a good (robust?) variogram model is available, kriging provides the estimation best representing the spatial distribution of the input data.

In the simplified kriging equation, the kriged estimate $Z_x^* = \sum w_i Z(x_i)$, where, w_i is the weight assigned the sample Z at location x_i and $\sum w_i = 1$.

Unlike other interpolation techniques (inverse distance power, splines, triangulation, etc.) the kriging weights are derived based on a data-driven weighting function making the sum of the weights equal to one, thereby reducing the effect of bias towards input sample values.

Should We Krige?

The decision of whether or not to use kriging will depend upon various factors such as spatial data distribution and structure, post kriging analyses, etc. If the input data are located in a near-perfect grid pattern, it is likely that kriging will produce similar estimates as other interpolation techniques, but may reduce the bulls-eye effect (as often seen in inverse distance square estimates). If input samples are clustered, then a declustering application should be used before variogram analyses. The kriging technique takes care of the cluster effect, whereas other interpolation techniques fail to do so.

The kriging technique, being a geostatistical technique, should be applied only if the underlying assumptions of second-order-stationarity are met, i.e., at a minimum the mean and variance of the sample data remain invariant in space. If the mean and variance of the samples do not meet these stationarity assumptions, it is advisable to apply an interpolation technique other than kriging.

The other very important factor to be considered before choosing a kriging interpolation technique is the homogeneity, i.e., the data represents one single homogenous domain. Such a domain can be characterized by lithology, geochemical characteristics and geological structures. Since geology is an integral part of geostatistics, much attention is required to understand the geology and identify individual domains. In Danie Krige's own words, "geology and geostatistics are the two inseparable parts of the same coin" and ". . . geology should form the foundation of any geostatistical analysis"[1].

The other basic requirement of using the kriging technique is having a sufficient number of data points. Even though there is no defined criterion for the number of samples to be used in the kriging process, most prominent authors have noted that a good quantity of data is likely to produce a more robust estimation. Empirically it is known that approximately 40 samples are needed to reasonably define and model a variogram, which can be used to produce an acceptable kriged estimate. Clustering (proximity) is also a consideration with the number of samples.

Pre Kriging Data Processing

It is essential to analyze the data and establish the statistical parameters so that a near-robust kriging profile can be designed for a reasonable kriging estimate. The pre-kriging data analyses are summarized below.

- The data set needs to be divided into subsets each representing a homogenous domain. Domains should to be defined based on geology, geochemistry and geological structures. A geological domain (lithology) divided by a fault should be considered as two domains.

Commonly the statistical analyses are combined into a group known as exploratory data analyses (EDA). The EDA should be done for data in each domain. The common statistical analyses in EDA include the following:

- *Spatial distribution of the data*: Often done by visual inspection of base maps of the samples.

- *Sample spacing*: Often done by summary statistics of the spacing between the composited drill-hole data. The summary statistics of an omni-directional variogram can also be used for this purpose. The decision made after these analyses helps to define the lag-distance in variogram analysis.

- *Spatial continuity:* An H-scatter plot can be used to assess this parameter. An H-scatter plot is an X-Y representation of the value of the first variable at the first sample location versus the value of the second variable (which can be identical to the first one) at the second sample location at a distance of separation d_i. The shape of the cloud of points spreads out as the spatial correlation between the two samples decreases.

- *Scatter plot:* A simple but an important tool for multivariate

[1] Krige, DG, 1999, Essential basic concepts in mining geostatistics and their links with geology and classical statistics" S. Afr. J. Geol, 102(2), 147 - 151

geostatistical analyses, leading to co-kriging.

- *Summary statistics*: The number of samples, the minimum, the maximum, the variability (variance and standard deviation), shape of the distribution with coefficients of skewness and kurtosis, and quantiles are commonly used. In multivariate cases, the correlation illustrated by quantile-quantile (Q-Q) plots between variables is useful. It is advisable to do summary statistics of the data at various locations within a single domain to verify the stationarity assumptions are met.

- *Histograms*: Histograms and cumulative frequency plots (CFP) are used in conjunction with the summary statistics in the EDA.

- *Principal component analysis (PCA)*: The PCA helps to do quick and near-simultaneous statistical analysis of several variables.

- *The search parameters:* The search parameters help to choose the input data for estimation of a particular block in a resource model. The search parameters used in kriging estimation are designed to reflect the spatial anisotropy and the spatial distribution of the data. Design of search parameters is one of the critical steps in kriging and should be done based upon the results of the EDA.

Advantages of Kriging

In this section the benefits of the kriging technique, in preference to other techniques are discussed. The advanced geostatistical techniques based on kriging used for risk analysis are also briefly discussed.

The properties of the elemental concentrations (variables) change with different types of deposits/ mineralization and space. The geology of a specific mineralization being unique requires special treatment for extracting the best predictive model (a kriged resource model). Over the years, the family of kriging techniques has provided unique solutions. The commonly used linear kriging techniques include ordinary kriging, simple kriging, and cokriging. The popular non-linear techniques include indicator kriging, for the estimation of discrete variables; lognormal kriging, for highly skewed distributions; disjunctive kriging, for non-linear problems; and factorial kriging analysis, to extract components of a model. Hence kriging provides flexibility to choose the technique that is most appropriate for a particular mineral deposit.

Kriging provides estimation of errors: kriging variance and block variance, along with the kriged estimates (Z_k). Kriging variance provides a basis for conditional simulation of possible realizations.

A uniquely designed kriging technique can be used to estimate confidence intervals of block estimates. This technique is based on the research work of Roth and Armstrong (1996)[2].

An advanced non-linear technique, uniform conditioning, is based on kriging and used to assess the probability of recoverable reserves. The uniform conditioning uses a discrete Gaussian model of change-of-support to assess the influence of the size of the selective mining unit (SMU) on grade-tonnage curves on a local scale.

Issues With Kriging

The kriging technique has many issues, which need special attention in order to understand the system and procedures.

Negative weights: It is possible that, due to clustering, some samples may receive negative weights. Negative weights may result in negative kriged estimates, which is highly undesirable. Similar results may occur if a highly structured variogram is used, ignoring the inherent randomness (nugget effect) in the structure. A negative block can be assigned a zero value to avoid problems in post kriging analyses, such as pit-optimization. Such results can be avoided if a more realistic variogram is used and/or data are de-clustered.

A deterministic technique: Kriging, being a deterministic interpolation

[2] Roth C., Armstrong Margaret, 1996, "Confidence intervals for local estimation: application to the Witwatersrand basin", in R.V. Ramani (ed.), 26th Proceedings of the Application of Computers and Operations Research in the Mineral Industry, APCOM'96, USA, Society for Mining, Metallurgy, and Exploration

technique, does not allow assessment of the risk of resource-estimation.

Search parameters: Search parameters play an important role in kriging. The search parameters should be designed with consideration of spatial structure and spatial distribution of the data in consideration. Small search radii may result in very conservative estimations as compared to very large search parameters. The later may result in over smoothing. A quantitative kriging neighborhood analysis (QKNA) was proposed by Vann et al (2003)[3] to design search parameters.

Exclusions: Kriging cannot be applied to many man-made data assemblages because the man-made data may not meet second order stationarity criteria. A tailing pond with frequently changing feed-points is such an example.

Conclusions and Comments

The popularity of kriging technique in mining industry is due to the realistic resource model that is created when kriging parameters are defined following careful exploratory data analysis. But kriging is a deterministic technique and hence insufficient for risk analysis on its own. On the other hand kriging forms the basis for advanced risk analysis techniques.

A kriged resource model of a mineral deposit is a predictive model and a representation of real world information. When dense samples become available, such as blast holes during production, an updated kriged model comes closer to reality accordingly. A kriged resource model created in an early stage of mineral deposit development may have blocks of larger size; such a model is good for long-term mine planning. With the development of the mining, the model should be reconciled with the blast hole information. For short term mine planning with blocks of SMU dimension, the resource model should incorporate the blast hole information and should be updated using results of prior reconciliation iterations.

Mineral resource exploration and extraction being a risky business, the advancement of new research on geostatistical risk analysis techniques, such as analysis of local recoverable reserves based on the support effect, adds to the importance of kriging technique in the mineral industry. It also indicates that more care must be taken before kriging is applied, so that a near-robust kriged estimate can be achieved. Understanding the fundamentals of the technique and process are important and require strong training in these areas. Many good text books are available to help build the foundation. Books such as the one by Armstrong (1998)[4] and by Isaaks and Srivastava (1989)[5] are easy to understand and provide guidance for kriging. Scientific research articles are valuable resources to update on new techniques being developed worldwide.

More recently, "time" is also being used as a continuous variable, which can be applied to environmental geochemistry related problems, where chemistry changes with time. More industry collaborated advanced research in geostatistics could lead to solving many problems that still exist with data information.

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[3] Vann, J., Jackson, S., Bertoli, O., 2003 "Quantitative Kriging Neighbourhood Analysis for the Mining Geologist." Proceedings - 5th International Mining Geology Conference. AusImm Publication Series 8/2003.
[4] Armstrong M, 1998, "Basic linear geostatistics", Berlin , Springer Verlag
[5] Isaaks, E., and Srivastava, R., 1989, "An Introduction to Applied Geostatistics": Oxford University Press, New York, 561 p.